Math 241 Calculus III (Prof. Honold) **Midterm Exam 1** 2023/10/25

Name: ____________________ Student No.: ____________________ Group A

For each of the following problems, find the correct answer (tick as appropriate!). No justifications are required. Each problem has exactly one correct solution, which is worth 1 mark. Incorrect solutions (including no answer, multiple answers, or unreadable answers) will be assigned 0 marks; there are no penalties.

1. The area of the triangle with vertices $(1,2,0)$, $(0,1,2)$, $(2,0,1)$ is

   ☐ $\sqrt{6}$ ☐ $\sqrt{3}$ ☐ $\frac{1}{2}\sqrt{3}$ ■ $\frac{3}{2}\sqrt{3}$ ☐ $2\sqrt{3}$

2. The height of the pyramid with the triangle in Question 1 as base and 4th vertex $(2,2,3)$ is

   ☐ $2/\sqrt{3}$ ■ $4/\sqrt{3}$ ☐ $8/\sqrt{3}$ ☐ $12/\sqrt{3}$ ☐ $24/\sqrt{3}$

3. For $\mathbf{A} = \frac{1}{2}\begin{pmatrix} -\sqrt{3} & -1 \\ 1 & -\sqrt{3} \end{pmatrix}$ the smallest integer $n > 0$ satisfying $\mathbf{A}^n = \mathbf{I}_2$ (the 2×2 identity matrix) is

   ☐ 3 ☐ 6 ☐ 8 ■ 12 ☐ 24

4. The distance between the lines $(0,1,1)+\mathbb{R}(1,-2,0)$ and $(1,1,0)+\mathbb{R}(0,1,-2)$ is

   ☐ $\sqrt{7/3}$ ☐ $1/\sqrt{21}$ ☐ 0 ☐ $\sqrt{21}$ ■ $\sqrt{3/7}$

5. The tangent to the twisted cubic $g(t) = (t,t^2,t^3)$, $t \in \mathbb{R}$ in the point $(-1,1,-1)$ intersects the plane $2x-y+z=3$ in the point

   ☐ $(1,1,2)$ ☐ $(1,1,1)$ ☐ $(1,0,1)$ ☐ $(1,-1,0)$ ■ $(0,-1,2)$

6. The minimum value of $f(x,y,z) = xy - yz + zx$ on the sphere $x^2+y^2+z^2=9$ is

   ■ -9 ☐ -6 ☐ -3 ☐ 3 ☐ none of the foregoing

7. For the helix $f(t) = (\cos t, \sin t, t)$, $t \in \mathbb{R}$ the unit normal vector $\mathbf{N}(\pi/4)$ is a positive multiple of

   ■ $(0,0,1)$ ■ $(-1,-3,\sqrt{2})$ ■ $(-3,-1,\sqrt{2})$ ■ $(-1,-3,0)$ ■ $(-3,-1,0)$

8. The arc length of the curve $g(t) = \left(t\cos t, t\sin t, \frac{1}{6}t^3\right)$, $t \in [0,2]$ is

   ☐ $\frac{15}{2}$ ☐ 42 ☐ $\frac{7}{6}$ ☐ $\frac{44}{3}$ ■ $\frac{10}{3}$

9. If $f\colon [0,3] \to \mathbb{R}^3$ satisfies $f(0) = (0,1,0)$ and $f'(t) = (t^2-1, 2t, t^2+1)$ then the point $f(3)$ is equal to

   ■ $(6,10,12)$ ☐ $(5,9,11)$ ☐ $(6,10,11)$ ☐ $(6,9,12)$ ☐ $(5,10,12)$

10. For a differentiable curve $\gamma = \gamma(t)$ in $\mathbb{R}^3$ the derivative $\frac{\mathrm{d}}{\mathrm{d}t}\frac{\gamma}{|\gamma|}$ is equal to

    ☐ $\frac{\gamma'}{|\gamma|}$ ☐ $\frac{|\gamma|\gamma' - |\gamma'|\gamma}{|\gamma|^2}$ ☐ $\frac{\gamma'}{|\gamma'|}$ ■ $\frac{\gamma'}{|\gamma|} - \frac{(\gamma\cdot\gamma')\gamma}{|\gamma|^3}$ ☐ 0

Time allowed: 45 min CLOSED BOOK ***Good luck!***